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1. **Quantum Entanglement and Bell’s Inequality Violation Simulation**

Objective:

Simulate the violation of Bell’s inequality using entangled qubits and compare classical vs quantum correlations.

Level: Undergraduate / Early Master’s

Skills: Basic linear algebra, quantum gates, probability theory, Python/Qiskit

What You'll Do:

#1 Create Bell states (e.g., ∣Φ+⟩=12(∣00⟩+∣11⟩)|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)∣Φ+⟩=2

#2 Simulate measurements at different angles
#3: Compute the CHSH value to show Bell’s inequality is violated

References:

- Nielsen & Chuang, *Quantum Computation and Quantum Information* – Ch. 2
- Qiskit Bell Test Tutorial
- Bell’s Theorem explained

2. **Quantum Teleportation Protocol with Fidelity Analysis**

Objective:

Implement the quantum teleportation protocol and measure the fidelity of the teleported qubit under different noise models.

Level: Undergraduate / Master’s

Skills: Quantum circuits, density matrices, quantum noise models

What You'll Do:

- Build the teleportation circuit in Qiskit
- Use the Aer simulator to add depolarizing or amplitude damping noise
- Analyze fidelity using statevector and density matrix simulators

References:

- Bennett et al. (1993) *Teleporting an Unknown Quantum State*, arXiv:quant-ph/9511030
- Qiskit Teleportation Lab
- Preskill Lecture Notes: Quantum Entanglement

3. **Building and Simulating the Quantum Fourier Transform (QFT)**

Objective:

Implement and visualize the Quantum Fourier Transform (QFT) circuit, and analyze its role in quantum algorithms.

Level: Undergraduate / Master’s

Skills: Complex numbers, quantum gates, phase shifts, Qiskit or Cirq

What You'll Do:

- Build QFT circuit for 2, 3, and 4 qubits
- Compare QFT outputs with classical Fourier Transform
- Integrate into small Shor’s algorithm simulations

References:

- Nielsen & Chuang – Ch. 5 (QFT)
- Qiskit QFT Tutorial
- Shor, P. (1994). *Algorithms for quantum computation: discrete logarithms and factoring*

4. **Quantum Circuit Optimization and Compilation Techniques**

Objective:

Study and implement techniques to minimize quantum circuit depth and gate count for standard algorithms (e.g., Deutsch-Jozsa, Grover).

Level: Master's

Skills: Circuit analysis, combinatorics, quantum gate synthesis

What You'll Do:

- Use Qiskit's transpiler to benchmark gate counts
- Apply optimization techniques (gate fusion, qubit mapping, commutation)
- Evaluate trade-offs between fidelity and circuit complexity

References:

- Qiskit Transpiler Docs
- Iten et al. (2016). *Quantum circuits for isometries* – arXiv:1602.02685
- Quantum Circuit Optimization Survey

🔹 5. **Simulating Multipartite Entanglement: GHZ and W States**

Objective:

Create and analyze GHZ and W states for 3+ qubits, and study their entanglement and decoherence behavior under noise.

Level: Advanced Undergraduate / Master’s

Skills: Quantum entanglement, noise modeling, simulation

What You'll Do:

- Build GHZ and W state circuits in Qiskit
- Use entanglement measures (e.g., von Neumann entropy, concurrence)
- Simulate behavior under different noise channels

References:

- Qiskit Entangled States Tutorial
- Dur et al. (2000). *Three qubits can be entangled in two inequivalent ways* – arXiv:quant-ph/0004088
- QuTiP for entanglement entropy: http://qutip.org