



FIRST/SECOND SEMESTER

Course Handout (Part II)

Date:.....

In addition to the part I (General Handout for all courses appended to the time table), this portion gives further specific details regarding the course.

Course No. : MATH G514
Course Title : Topics in Analysis
Instructor-in-charge :
Instructors :

1. Course Description: In this course, students will learn some of the implications of completeness and compactness of metric spaces, including the Baire-category theorem and Arzela-Ascoli's theorem, Measure and integration on general measure spaces, Radon-Nykodym theorem, and its implications in probability theory, the fundamental theorem of Lebesgue integration, product spaces and Fubini theorem, Fourier transform as a linear operator, Fourier inversion formula, Fourier-Plancheral theorem and their implications.

2. Scope and Objective of the Course: This course is meant for PhD scholars before they embark on research in a specific area. Basic notions and theorems will be introduced and their significance in the context of other subjects in mathematics will be explained in the classes. Some of the details are expected to be filled by students as home-assignments or by presenting as seminars.

3. Text Books:

- (1) N.L. Carothers, Real Analysis, Cambridge University Press, 2006.
(2) M.T. Nair, Measure and Integration: A First Course, Taylor and Francis, CRC Press, 2020.

4. Reference Books:

- (1) W. Rudin, Principles of Mathematical Analysis, International Student edition. 3rd Edition, 1976.
(2) C.C. Pugh, Real Mathematical Analysis, Springer, 2002.
(3) G. De Barra, Introduction to Measure Theory, Van Nostrand Reinhold, 1974.
(4) H.L. Royden, Real Analysis, 3rd Edition, Macmillan, 1988.

5. Lecture Plan:

Table with 3 columns: Lecture Numbers, Module, Topics to be covered

Lectures: 1-6 (TB-1; Ch: 3-5)	1. Review of basic notions in metric spaces	Definitions and examples of metric spaces, open and closed sets, closure of a set, limit points of sets. Continuous functions between metric spaces, completeness and compactness.
Lectures: 7-11 (TB-1: Ch: 7, 9)	2. Completeness, Weierstrass approximation theorem and Baire-category theorem	Cauchy completion and completion using completeness of spaces of bounded functions; Weierstrass approximation theorem; Nowhere dense sets; First and second Category of metric spaces; Baire-category theorem and its implications.
Lectures: 12-17 (TB-1: Ch: 8, 10, 11)	3. Compactness and Arzela-Ascoli's theorem	Totally bounded and compact metric spaces; The set $C(X)$ of continuous functions on a compact metric space $X$ ; Totally bounded subsets of $C(X)$ with respect to sup-metric; Equi-continuous family of functions; Arzela-Ascoli's theorem on compactness of subsets of $C(X)$ .
Lectures: 18-23 (TB-2: Ch: 2)	4. Review of Lebesgue measure on $\mathbb{R}$	Lebesgue outer measure and Lebesgue measure, and their basic properties; Set of all Lebesgue measurable sets as a $\sigma$ -algebra.
Lectures: 24-32 (TB-2: Ch: 3,4, 5)	5. Integration over general measure space	General measure space and measurable functions; Integrals of simple and positive measurable functions on a general measure space; Monotone convergence theorem; Integrals of complex measurable functions; Dominated convergence theorem; Radon-Nykodym theorem, and its implications in probability theory; $L^p$ -space as a Banach space.
Lectures: 33-37 (TB-2: Ch: 5)	6. Fundamental theorem of Lebesgue integration	Absolutely continuous functions; Functions of bounded variation; Indefinite integral with respect to Lebesgue measure; Absolute continuity of indefinite integral; Fundamental theorem of Lebesgue integration.
Lectures: 38-41 (Text-2: Ch: 6, 7)	7. Product Measure and Fourier Transform	Product measure and Fubini theorem; Definition and basic properties; Fourier transform as a linear operator; Fourier inversion formula; Fourier-Plancherel theorem.

## 6. Evaluation Scheme:

EC No.	Evaluation Component	Duration	Weightage (%)	Date, Time	Remarks
1	Mid-sem	90 Min.	30	***	***
2	Tutorial Test/Quiz/ Assignments/Seminars	***	30	***	***
3	Comprehensive Exam	3 Hours	40	***	***

7. **Problems:** Students are strongly advised to work out the problems in the textbooks and do similar problems from the reference books.

8. **Chamber Consultation Hour:**  
Notices:

**Instructor-In-Charge**

10. **Make-up Policy**

**MATH G514**